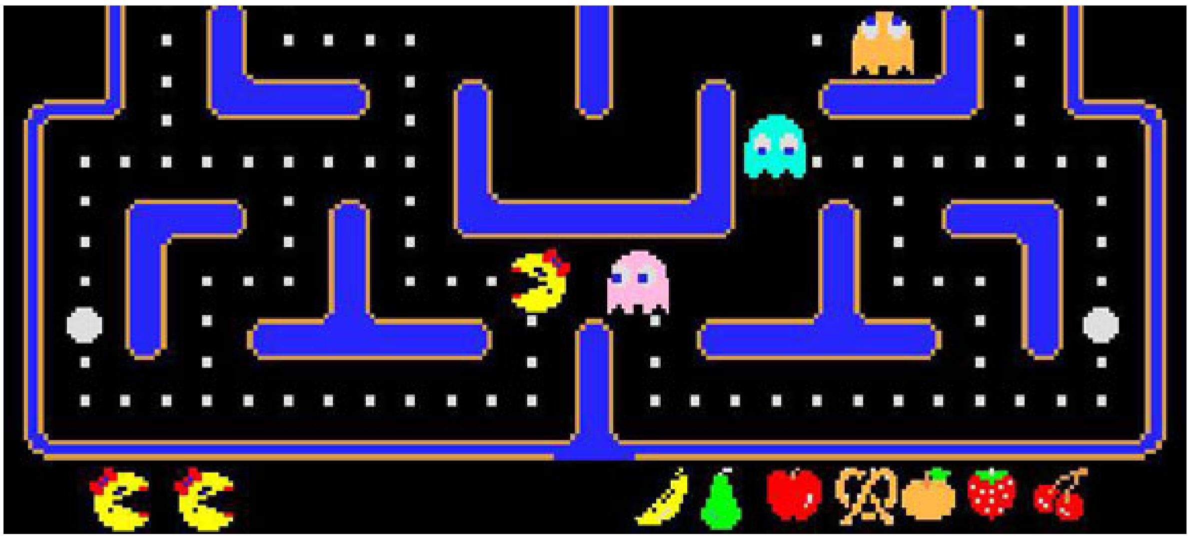
How Did They Make Ms. Pac-Man?



The Situation

You want to make a video game similar to the one below and are beginning by animating Ms. Pac-Man.

The Challenge(s)

- How can you describe Ms. Pac-Man's movements?

Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- What movement did Ms. Pac-Man make?
- What was the very first thing Ms. Pac-Man did?
- Does anyone have the same answer but a different way to explain it?
- How did you reach that conclusion?
- How can you demonstrate what you are saying is correct?

- What assumptions are you making?

Consider This

This lesson provides a real-life context for transformations including rotations, reflections, and translations which are the foundation for how the Common Core State Standards require students to understand congruence and similarity. Rather than begin the lesson by defining the terms and identifying them in the game, the goal is to let students initially describe the movements in their own words and then guide them towards a mathematically precise definition.

I made assumptions about what students would say and have created animated videos to show the imprecision in students' statement. Specifically, if you begin by showing students the video game movie clip above and ask them, "What movement did Ms. Pac-Man make?" my guess is that they will say that "Ms. Pac-Man first moved to the right, then up, then right, then …" What they are imprecisely describing are translations and you can help them "use clear definitions in discussion with others and in their own reasoning" (Math Practice 6) by telling them that "Mathematicians calls those types of movements translations." Then show them the video below:

You may need to switch back and forth between the original video and this new video to compare the two sets of movements. What I hope they realize is that while it is true that Ms. Pac-Man translated across the screen (we will discuss this in more precise terms later), that was not the only movement Ms. Pac-Man made. Ask them "What was the very first thing Ms. Pac-Man did?" and expect to hear answers like "She looked the other way" or "She turned to the right." Again you can tell them that "Mathematicians call those types of movements reflections." Then show them the video below (NOTE: YouTube cut off the first second so if may look like Ms. Pac-Man begins reflected even though she doesn't):

Again, you may need to switch back and forth between the original video and this new video to compare the two sets of movements. Something important to note is that when Ms. Pac-Man gets to the top right of the screen, she starts doing both rotations and reflections where as in the beginning she is only doing one transformation at a time. In the video below I am only showing translations and reflections (no rotations) so she does one reflection at the very beginning and another towards the top right of the screen.

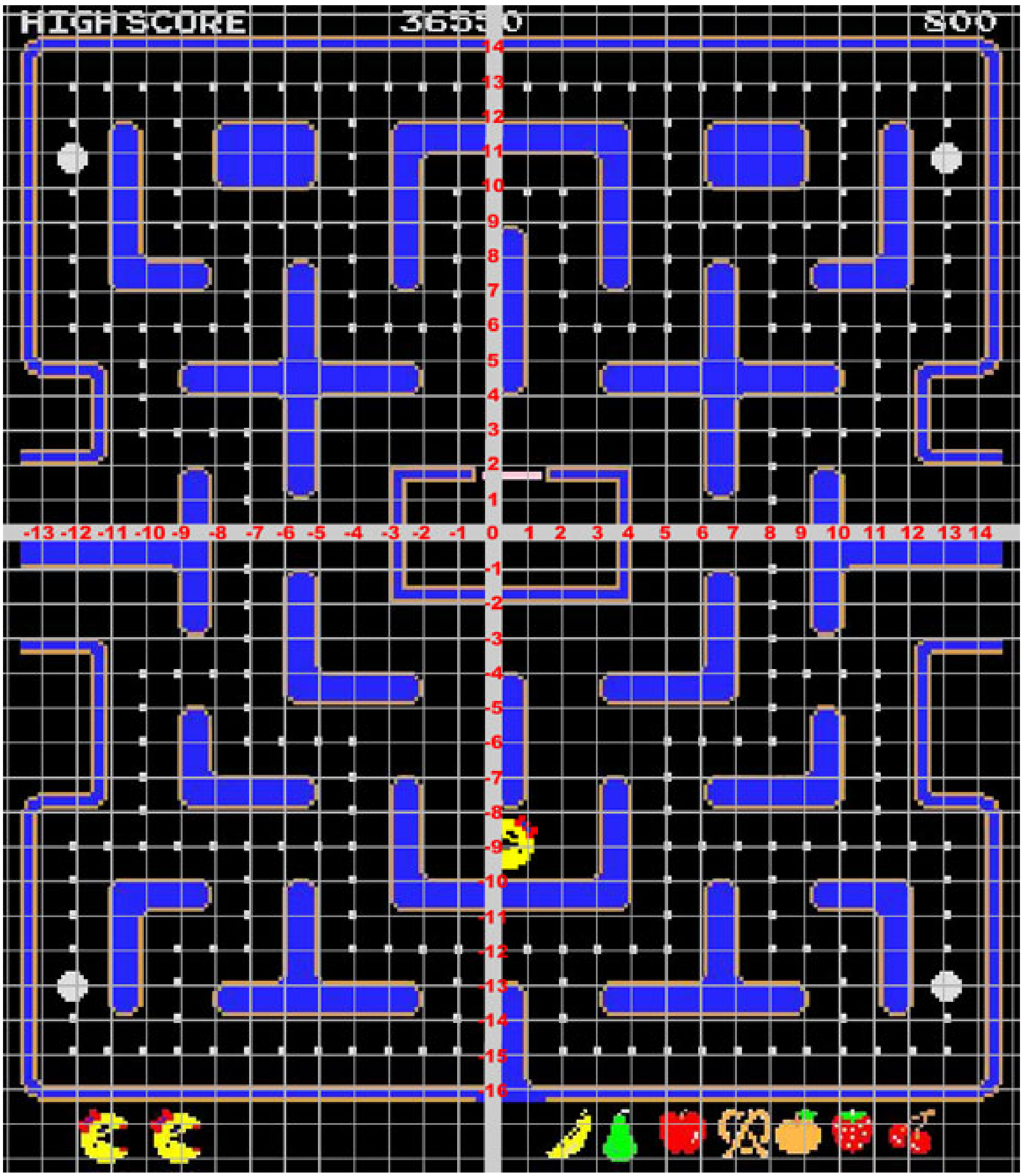
At this point they will most likely realize that Ms. Pac-Man turns in addition to the other movements. Similarly you can tell students that "Mathematicians call those types of movements rotations." Then show them the video below:

By now students will realize that Ms. Pac-Man is making a series of translations, reflections, and rotations but we need to increase our mathematical precision and list them out. To help students realize this lack of precision ask them questions which should illustrate our communication problem like:

- How far did she translate?
- What unit are we measuring this in?
- Which way did she reflect?
- How much did she rotate?

Hopefully students will realize that to answer these questions (and at least the first one) we will need a common unit of measure. Students may something like, “She moved up 5 dots ” but what about in the beginning where there are no dots? When students are realizing they need some structure to have a meaningful conversation, show them the video below:

Have students to create a list of these movements on a sheet of paper. You will likely have to play the clip above many times and also give them printed copies of the images below. The grid helps give them the coordinates and the two Ms. Pac-Man pictures are very helpful in demonstrating the rotation/reflection combinations:



Once students have come up with their list of transformations, have a conversation about them as students may have incorrect answers, a different right answer, or multiple ways to explain it. These questions may be especially useful:

- Does anyone have the same answer but a different way to explain it?
- How did you reach that conclusion?
- How can you demonstrate what you are saying is correct?
- What assumptions are you making?

I went through the video and made my own list of transformations. Here is what I have (included as a PDF in the “Download files” link):

1. Start at (0.5, -9)
2. Reflection across line x = 0.5
3. Translation 1.5 units right to (2, -9)
4. Rotation 90° counterclockwise
5. Translation 3 units up to (2, -6)
6. Rotation 90° clockwise
7. Translation 6 units right to (8, -6)
8. Rotation 90° counterclockwise
9. Translation 9 units up to (8, 3)
10. Rotation 90° clockwise
11. Translation 3 units right to (11, 3)
12. Rotation 90° counterclockwise
13. Translation 3 units up to (11, 6)
14. Two possibilities (UP THEN LEFT)
    1. Reflection across line x = 11 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = 11
15. Translation 3 units left to (8, 6)
16. Two possibilities (LEFT THEN UP)
    1. Reflection across line x = 8 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = 8
17. Translation 3 units up to (8, 9)
18. Two possibilities (UP THEN LEFT)
    1. Reflection across line x = 8 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = 8
19. Translation 3 units left to (5, 9)
20. Two possibilities (LEFT THEN DOWN)
    1. Reflection across line x = 5 AND Rotation 90° clockwise
    2. Rotation 90° counterclockwise AND Reflection across line x = 5
21. Translation 3 units down to (5, 6)
22. Two possibilities (DOWN THEN LEFT)
    1. Reflection across line x = 5 AND Rotation 90° clockwise
    2. Rotation 90° counterclockwise AND Reflection across line x = 5
23. Translation 3 units left to (2, 6)
24. Two possibilities (LEFT THEN UP)

1. Reflection across line x = 2 AND Rotation 90° counterclockwise
2. Rotation 90° clockwise AND Reflection across line x = 2

25. Translation 4 units up to (2, 10)
26. Two possibilities (UP THEN LEFT)

    1. Reflection across line x = 2 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = 2

27. Translation 3 units left to (-1, 10)
28. Two possibilities (LEFT THEN DOWN)

    1. Reflection across line x = -1 AND Rotation 90° clockwise
    2. Rotation 90° counterclockwise AND Reflection across line x = -1

29. Translation 4 units down to (-1, 6)
30. Two possibilities (DOWN THEN LEFT)

    1. Reflection across line x = -1 AND Rotation 90° clockwise
    2. Rotation 90° counterclockwise AND Reflection across line x = -1

31. Translation 3 units left to (-4, 6)
32. Two possibilities (LEFT THEN UP)

    1. Reflection across line x = -4 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = -4

33. Translation 3 units up to (-4, 9)
34. Two possibilities (UP THEN LEFT)

    1. Reflection across line x = -4 AND Rotation 90° counterclockwise
    2. Rotation 90° clockwise AND Reflection across line x = -4

35. Translation 5 units left to (-9, 9)

To be honest, it got much trickier for me starting with Transformation #14. I had to make Ms. Pac-Man drawings and prove to myself that what I wrote down was correct. It is certainly easy to get mixed up as this requires strong visual/spatial skills.

A few notes:

- I initially put the ALL CAPS phrases in when I started noticing a pattern in the transformations. I intended on deleting them but ultimately decided to keep them in as they might be helpful to you.
- I listed “Two possibilities” but technically there are infinite possibilities:
    - You could add multiples of 360° to the rotation (so instead of 90° you could have 450°) and that would take you to the same place.

- Also, rotating 90° in one direction is the same as rotating 270° in the other direction.

Content Standard(s)

- CCSS 8.G.1 Verify experimentally the properties of rotations, reflections, and translations.
- CCSS 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- CCSS 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- CCSS 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- CCSS G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- CCSS G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- CCSS G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Source(s)

- Ms. Pac-Man

Download