

### Log Ride

At theme park has a log ride that can hold 12 people. They also have a weight limit of 1500 lbs per log for safety reason. If the average adult weighs 150 lbs, the average child weighs 100 lbs and the log itself weighs 200, the ride can operate safely if the inequality

$$150A + 100C + 200 \leq 1500$$

is satisfied (A is the number of adults and C is the number of children in the log ride together. If 4 adults are already seated, what is the maximum number of children that may safely ride with them?

## Commentary

In this instructional task students are given two inequalities, one as a formula and one in words, and a set of possible solutions. They have to decide which of the given numbers actually solve the inequalities. There are two solutions presented, the first is the more natural one, where we simply substitute the given values and check if the inequality is satisfied. In the second solution, we actually solve the inequality first and then see if the given values are in the solution set. While this seems unnecessary, it is the approach many students will use because they have often been "conditioned" to solve any inequality or equality they encounter. A class discussion could clear up when it is necessary to actually solve an inequality and when it is easier to use substitution. If we were given a large number of possible solutions to check, solving would be the preferred solution method, while for a small number, substitution is faster.

This task provides a way to see if students understand the meaning of a solution to an inequality (or equation), namely values of the variables that make the inequality (equation) true. Often students see solutions only as the result of solving an inequality or equation. This idea is an important one, since for some equations it is possible to check if a given number is a solution, while it would be hard or impossible to solve, e.g. higher degree polynomials, equations involving trigonometric or exponential functions.

A natural extensions for this task would be to ask students to find other groups of adults and children that can or cannot ride together, either because of the number of people allowed or because of the weight limit. Some combinations are not possible. For example, it is not possible to find a group of children that satisfies the number requirement but does not satisfy the weight requirement.

### Solution: By Substitution

As there are five different groups waiting to ride, we can substitute each group's number of children into the weight equation for  $C$ , using  $A = 4$  as there are already 4 adults seated in the log. For group one, we get

$$150(4) + 100(4) + 200 = 1200 \leq 1500$$

So, group one can safely ride the ride. Using this technique with the other four groups, we find that group two, group four, and group five can all safely ride the ride both in weight and in number-of-riders. We find that group three exceeds the number-of-riders limit, as there will then be 13 people on the log, as well as the weight limit.

$$150(4) + 100(9) + 200 = 1700 \not\leq 1500$$

We can also observe that group one together with group two would be able to ride the log ride.

$$150(4) + 100(4 + 3) + 200 = 1500$$

and

$$7 + 4 < 12$$

### Solution: Solving the Inequality

While this approach is not expected of 6th graders, students who are ready for this approach may try it in 6th grade. This solution approach is expected in 7th grade.

First, using the given equation, we have that  $A = 4$  as there are 4 adults already seated. By substitution, we can solve for  $C$  and see how many children can safely ride based solely on the weight limit.

$$\begin{aligned} 150(4) + 100C + 200 &\leq 1500 \\ 600 + 100C + 200 &\leq 1500 \\ 100C &\leq 700 \\ C &\leq 7 \end{aligned}$$

So, up to 7 children will satisfy the weight limit inequality. Since  $7 + 4 < 12$  the number-of-riders limit is also satisfied, and so we find that 7 is the maximum number of children that can safely ride the ride with the 4 adults. Thus, group three is the only group that cannot safely ride in the log. Note: We can observe that group one and group two would be able to ride the log ride together with the 4 adults.