## Building A Winning Die

Activity: For this activity, you will need a partner and a die. You will be having a "rolloff" competition. You both roll your die at the same time. The person with the larger number gets a point. If you have the same number, record this as a tie. Keep track of how many points each of you has, as well as how many ties there are on the table below. Roll a total of 36 times. The person who has more points at the end will move up towards the winners table, and the player who lost will stay at his/her table.


Was this game fair? Why or why not?

Analyzing the competition: To determine if this was a "fair" competition, fill in the chart of all possible match-ups between the two die, A and B. In each box, record which die will win by writing A or B , or if it's a tie, write T for tie.


1. How many times should die A win? $\qquad$
2. How many times should die B win?
3. Does one person have a better chance/ probability than the other in this competition?
4. Why did some students win twice if everyone had an equal chance of winning?

Strategizing to Win:
We will be having a new tournament with the same rules as before. This time, however, you make your own die. Your die may have any digit from 0 to 6 on each face, but the sum of the six numbers on the faces must be 21 .

Competing in the Tournament: The tournament runs the same way as above: each round consists of 36 rolls. For each student you compete against, use tally mars to record you wins, your partner's wins and ties. Then, analyze who should have won by completing the table and calculating the probability of each person winning.
Game 3:
My Wins Partner Wins $\quad$ Ties


What is the probability that your die would win on a single roll? $\qquad$
What is the probability that your partner's die would win? $\qquad$
Which die would win more often in the long run? $\qquad$



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IMP Activity: Building a Winning Die
Game 5: $\quad$ Pr Partner Wins Ties


What is the probability that your die would win on a single roll? $\qquad$
What is the probability that your partner's die would win? $\qquad$
Which die would win more often in the long run? $\qquad$

## Analysis

1. Does the person with the greater probability of winning always win when you play? Why or why not?
2. What is the difference between experimental probability and theoretical probability?
3. How did the difference impact you in the games?
4. Write down the digits of the person who won the most in the class. Did he/she have the best die? Use the table below to compare your two dice and determine if you had a great theoretical probability of winning. Winning Digits: $\qquad$

| \#'s <br> on <br> die |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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5. Could you design a die (that still uses only the digits $0-6$ and has a sum of 21) to BEAT the best die? Use the tables below to help you plan how to build a die to be the winner.


## Teacher Directions

## Materials:

6 -sided dice ( 1 per person)
1 -inch colored cubes (1 per student)
Circle Sticky dots (6 per student)

## Objective:

Students will compete rolling standard dice to determine which has the larger number more often. They will analyze the game to determine if it was fair. They will then create their own die with a sum of 21 and compete against other students' dice to compare and understand the concept of experimental and theoretical probability.

## Directions:

Set up the class into table/groups of 4 . Within each group of 4 , have students decide who they will be playing against (each student needs 1 partner to play against). Pass out the activity sheet to each student. Explain the directions and ensure that all students can tell you that they will complete 36 rolls, each time recording a tally mark for the person who had the larger number. Pass out a die to each student (and if needed, show them how to set up a "goal" to roll into. Set the timer for 90 seconds for students to complete their 36 rolls. Ask each pair to determine who won. Now, explain how the competition will work. You need to determine the flow of tables so that there is a "losers or bottom" table and then a progression up to the winners table. Everyone who won needs to raise their hand and then take their paper, pencil and die and move UP to the next table, with this exception: if you win while at the top table, you stay and if you lose at the top table, you go down to the bottom table. Note: Two people should be moving from each table! Once everyone has moved, have them repeat the exact same game with a new partner.

After game \#2, have the students think about, and then record, "was this game fair?" Call on a few students to share their thinking and then direct them to the section "Analyzing the competition". Explain how the table works to show all possible outcomes (which will allow us to "prove" if the game was fair, as you did with the tree diagram for 2 coins, 3 people). Ask what the result is if Die A is a 1 and Die B is also a 1. Record a T in that box to represent a Tie. Then ask what happens in Die A is a 2 and Die B is a 1. Record a $B$ in that box to show that $B$ would have won. Have the students continue in this process to complete the table. Note: Many students will notice patterns for completing this table; encourage them to share once all have completed!

Ask the students to use the table to determine the probability of Die A and Die B winning. Give them 2 minutes to complete the questions at the bottom of page 1. Once you agree that the game is fair in that each person has an equal chance of winning, make sure students can explain why some students still won. Write the word "Experimental Probability" on the board as well as "Theoretical Probability". Guide the students to see that while they should have tied (Theoretically, on paper), experimentally, some people win. If you want, use this opportunity to explain why Las Vegas and places like it make money!

Now that everyone agrees you all have an equal probability of winning, the game is less fun, so explain that they will get a chance to make their own die. Explain the rules under "strategizing to win". They can use any digit from 0 to 6 , but the SUM must be 21. Pass out sticky dots and blank cubes and allow each student 5 minutes to plan and make their die.

Repeat the same process of competing, but this time students use their own die. IMPORTANT NOTE: before starting, have each student check to ensure their partner's die has a sum of 21. After EACH competition, they need to use the table to compare who should have won (this practice also helps them analyze 2 dice!). Have the students compete 2 more times, stopping after each game to have the winners (experimental) move "up".

Once you have completed the 5 "games", give the students 5 minutes to complete analysis questions 1-3.

## Challenge/Extension

Find a student at the top who won all 3 times with his/her created die. List the digits of this die on the front as ask the class if they think this person should have won or if this person got lucky. To know the answer, have each student compare their die to these 6 digits using the table under \#4.

Record a few sets of numbers that theoretically should have won. Ask the students if these numbers represent the best die. Have them either compare theirs to one of these OR try to create a die to "beat" this one. Note: There is NOT a best die, but you can always build a die to tie or beat the "winning" die. See how well your students are doing with this activity and if they are keeping up and interested, create a diagram to show which numbers are beating which other sets and have them try to create (in theory) a die to beat each winner. Show them how the transitive property does NOT hold in this case: just because A beats B and B beats C does NOT mean A will beat C.

