N.RN Operations with Rational and Irrational Numbers

a. Complete the addition table.

	5	1/2	0	$\sqrt{2}$	$-\sqrt{2}$	π
5	10	5.5				
1/2						
0						
$\sqrt{2}$						
$-\sqrt{2}$						
π						

b. Complete the multiplication table.

	5	1/2	0	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	π
5	25	2.5				
1/2						
0						
$\sqrt{2}$						
$\frac{1}{\sqrt{2}}$						
π						

c. Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true?
   i. The sum of a rational number and a rational number is rational.

   ii. The sum of a rational number and an irrational number is irrational.

   iii. The sum of an irrational number and an irrational number is irrational.

   iv. The product of a rational number and a rational number is rational.

   v. The product of a rational number and an irrational number is irrational.

   vi. The product of an irrational number and an irrational number is irrational.

Commentary

This task has students experiment with the operations of addition and multiplication, as they relate to the notions of rationality and irrationality. As such, this task perhaps makes most sense after students learn the key terms (rational and irrational numbers), as well as examples of each (e.g., the irrationality of $\sqrt{2}$, π, etc.), but before formally proving any of the statements to be discovered in this task. Discussion of such proofs is taken up in other tasks.

These conjectures are likely best discussed in small groups and/or with the whole class, and so is best used in instructional, rather than assessment-based, settings. The discussions generated by student conjectures will likely yield productive insights into the nature of sums and products of real numbers leading eventually to the explanations sought in content standard N.RN.3, preparing them for the formal statements of these results. Note that some of these decisions, e.g., the irrationality of $\pi + \sqrt{2}$, are well beyond the scope of high school mathematics, but this does not preclude students from being able to answer the always/sometimes/never questions being asked.

Solution: Solutions

Solution to part (c):

i. The sum of a rational number and a rational number is rational.

Always true.

ii. The sum of a rational number and an irrational number is irrational.

Always true.

iii. The sum of an irrational number and an irrational number is irrational.

Only sometimes true (for instance, the sum of additive inverses like $\sqrt{2}$ and $-\sqrt{2}$ will be 0).

iv. The product of a rational number and a rational number is rational.

Always true.

v. The product of a rational number and an irrational number is irrational.

Not true -- but almost! This holds except when the rational number is zero.

vi. The product of an irrational number and an irrational number is irrational.

Only sometimes true (for instance, the product of multiplicative inverses like $\sqrt{2}$ and $\frac{1}{\sqrt{2}}$ will be 1).